Capacity Expansion of Optical Networks with WDM Systems Under Demand Uncertainty

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Abstract— **In this paper we develop models and methodologies for the wavelength division multiplexing and optical cross-connect routing and provisioning problem under demand uncertainties. Uncertain demands are modelled by a set of scenarios with known probabilities. Our design algorithm relies on the application of the scatter search optimization metaheuristic. A realistic test case having 17 links, 12 nodes and 19 demands is used to illustrate the proposed procedure.**

Keywords— **Network Planning, Scatter Search, Robust Optimization**

I. Introduction

The problem that we address in this paper is a real world problem that results from the need to expand capacity of telecommunication networks built with fiber optics technology in the presence of uncertain information. When deterministic information is considered, given a network physical topology and the estimate of the point-to-point demand traffic, the problem is to determine the routing for each demand and the least-cost WDM and OXC equipment configuration required to support the routes. This problem is modelled as a mixed integer problem (MIP).

Wavelength Division Multiplexing is the transmission of multiple laser signals at different wavelengths (colors) in the same direction, at the same time and over the same strand of fiber. WDM with more than eight frequencies, called Dense Wavelength Division Multiplexing (DWDM), which creates multiple bi-directional virtual fibers per physical fiber, currently enables a low cost per bit. DWDM solves the bandwidth bottleneck resulting from growth in data traffic, because it is an emerging technology that increases transportation capacity while preserving optical fiber equipments previously installed. Hence, DWDM provides carriers the flexibility and scalability they need to deploy capacity when and where it is needed.

OXCs are small space-division switches that can switch an optical signal from one wavelength to another on multi-fiber WDM systems or on a single fiber.

Algorithms and model formulations have usually assumed that the data for the given problems are known accurately. However, this is not true in most of real applications due to measurement errors and other reasons. This is clear when data represent information about the future like traffic demands and product costs that can not be known with certainty. There are several ways to take into consideration the uncertainty in making optimal decisions. The usual techniques to deal with optimization problems with uncertainty in the data are sensitivity analysis, fuzzy optimization, stochastic programming and robust optimization.

One of the oldest techniques to model uncertainty in an optimization problem is the Sensitivity Analysis, which study the way the optimal solution changes after a slight modification in the data. Since the beginning of linear programming, a half century ago, sensitivity analysis has been a part of the field of post-optimality in the theory, the implementations and applications [7], [1]. As much as possible, these foundations have been extended to nonlinear, integer, stochastic, multicriteria, and other mathematical programming, though it is considered that those advances have so far not provided as rich a body of knowledge [23].

Fuzzy optimization uses the fuzzy set technology [24], [8], [5] and techniques to deal with uncertainty. A fuzzy optimization problem is an optimization problem where some of its components are uncertain and given by a membership function. The usual methods in fuzzy linear programming consider mainly fuzziness in the cost function, in the coefficients and in the inequalities of the constraints that can be verified in several degrees. When uncertain data are given by optimistic, pessimistic and intermediate values, triangular fuzzy numbers are a straightforward manner to deal with the problem.

Stochastic Programming is a framework for modelling optimization problems with uncer-

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tainty where probability distributions of data are known or can be estimated. The goal is usually to find a solution that is feasible for all (or almost all) the possible data instance and maximize the expectation of some function of the decisions and the random variables. A basic idea in most of methods in Stochastic Programming is the concept of recourse, that is the ability to take corrective actions after a random event has taken place. The simplest examples are the two-stage problems, in which some decision variables are fixed before some events occur and other decisions are taken after the events. The decision maker takes some actions in the first stage, after which a random event occurs affecting the outcome of the first stage decisions. A recourse decision can then be made in the second stage that compensates for any bad effects that might have been experienced as a result of the first stage decision. The optimal decision for such a model consists of a single decision for the first stage and a collection of recourse decisions defining which second-stage decision should be made in response to each random outcome. These problems are extended to multi-stage problem [2], which is closely related to multi-stage decision analysis, Markov decision process, stochastic control theory and dynamic programming. Solution approaches to stochastic programming usually consist in obtaining a deterministic equivalent optimization problem that is solved by known techniques. These problems are typically very large scale problems and of a different type. Several survey articles [3] and [21], and books [10], [4], [17] are dedicated to stochastic programming. The bibliography by van der Vlerk includes more than 3500 references [22].

Other techniques to deal with optimization problems that involve uncertainty are scenario-based, in which the scenarios correspond to the possible realizations of the uncertainty [19]. In practice, the uncertainty ranges from a few scenarios up to a precise joint probability distribution of all the random variables or data involved in the problem, where each possible scenario has its corresponding probability of occurrence. Uncertainty in key data is usually characterized by a probability distribution. The use of scenarios as a tool for modelling uncertainty has the advantage of not requiring knowledge of the underlying probability distributions associated with the random variables.

Robust optimization [16] belongs to the family of scenario-based optimization techniques. The main feature of robust optimization (RO) formulations is the flexibility to define the tradeoff between *solution robustness* and *model robustness*. A feasible solution to the problem is termed *robust solution* if it remains "close" to optimal for any realization of the scenario. The solution is also robust with respect to feasibility if it remains "almost" feasible for any realization of the scenario. Model robustness can be measured, for example, by the expected value of the infeasibility. Measuring the deviation of the proposed solution to the scenario-optimum gives an idea of its solution robustness.

II. The WDM and OXC Routing and Provisioning Problem

WDM technology and its related equipment such as OXC have several advantages when considering the increase of capacity of an existing optical network [18]. We highlight these advantages in our problem description. In order to increase the capacity of a network at a minimum cost, it is necessary to decide where to place WDM and OXC systems and how to route the traffic within the resulting network. We assume that a network design exists and that our problem consists of adding capacity to the current network in order to carry a set of demands while satisfying technology constraints.

Each WDM system must originate and terminate at an OXC or DCS (Digital Cross-Connect Signal) port. Also, an OXC or DCS port is needed to add or drop traffic at the origin and destination of each demand carried by the network. The OXC and DCS ports are bidirectional. The installed base of DCS machines generally lacks OC-48 ports, so that each DCS OC-48 port may physically consist of 48 DS3 ports. Since we will be modelling at the OC-48 level, we can consider OXC and DCS equipment to be functionally equivalent.

The deterministic optimization problem deals with a set of point-to-point demands that the existing optical network is incapable of absorbing. Associated with each demand is an origin node o, a destination node d, and a bandwidth requirement R_{od}, expressed in OC-48 units. Optical fiber joining pairs of nodes is used to route demands through the network. Each demand can be routed either entirely on one or more bare fibers, over one or more channels of a WDM system or it can be switched from a WDM to another through OXCs. The goal of the network planner is to minimize the total cost, that is, the cost of additional fiber, WDM systems and OXC equipment.

In our context, the physical network design (i.e., the set of exiting links) is given, which limits where additional optical fibers and WDM systems can be placed. The notion of a segment is used to represent a direct connection between an origin and a destination using glass-through nodes. Glass-through nodes for a segment do not use OXC systems and therefore traffic cannot be added or dropped between the origin and the destination. Each OC-48 unit uses two bare fibers or a channel of a WDM system. For convenience, we refer to the capacity required for an OC-48 unit as a channel, regardless of whether a pair of fibers or a channel of a WDM system is actually used. All links within a segment must be capable of carrying the same amount of traffic in order to satisfy the demand from its origin to its destination. In an optimal network design, each segment follows a least-cost path (with respect to fiber cost) from origin to destination. Once an OXC is reached, wavelengths and fibers can be rearranged. Therefore, the capacity constraints on each segment are simply that enough fiber and WDM equipment must be available on the segment to handle the number of OC-48 units assigned to it. Each individual link must have enough channel capacity to cover all demands routed over segments that uses it.

Kennington, et al. [11] formulate and solve a similar problem with demand uncertainties. In their work, however, they do not consider OXC equipment or glass-through nodes, and therefore they don't use the concept of segments. Other network planning problems with uncertainty in key data have been considered in [13], [9], [14] and [20].

III. The Models

In this section, we present a mixed integer programming (MIP) formulation of the basic provisioning problem. We use an arc-path model that determines the equipment required to route a set of point-to-point demands through the network for a given scenario. In addition, a robust optimization model is presented that determines the equipment needed when all scenarios are considered simultaneously. Model robustness is measured by the expected value of the infeasibility.

A. Data

The network topology is represented as a graph $G = (N, E)$, where N denotes the set of nodes and $E \subseteq N \times N$ denotes the set of segments. Therefore, in our formulation, links and segments are equivalent in that they represent a direct connection between two points. The cost of using an individual link or a segment is correctly computed in the objective function. For each $n \in N$, An denotes the set of segments adjacent to node n. The origin/destination node pairs corresponding to the point-to-point demands are given by $D \subseteq N \times N$. For each $(o, d) \in D$, R_{od} denotes the bandwidth requirements in OC-48 units and J_{od} denotes the set of possible paths from o to d that can be used to route this demand. Since the set of paths used for each demand may not consist of all the possible paths from o to d, the formulation described in this section may be used as a heuristic model for the provisioning and routing problem.

A.1 Cost Input Data

C^F_e = cost of a fiber on segment e (sum of costs per link along that segment).
C^W_e = cost of a WDM unit on segment e.
C^O = cost of an OXC unit.
C^c = channel cost of a WDM unit.
C^p = port cost of an OXC unit.

A.2 Capacity Data

M^W = capacity of a WDM unit.
M^O = capacity of an OXC unit.

A.3 Existing Infrastructure

g_e = spare WDM channels on segment e.
h_n = spare OXC ports at node n.

B. Decision Variables

$x^{od}_p = 1$ if demand (o, d) is routed on path p and 0 otherwise.
f_e = number of stand-alone (no WDM) fiber pairs on segment e.
w_e = number of WDM units on segment e.
v_e = number of channels in the WDM systems installed on segment e.
y_n = number of OXC units installed at node n.
u_n = number of ports in the OXC systems installed at node n.

C. Objective function

The following objective function, to be minimized, is the design cost which is the sum of stand-alone fiber costs (first term), WDM costs (second term) and the OXC costs (third term), when only one type of WDM and OXC systems is used.

$$\min \sum_{e \in E} 2C^F_e f_e + \sum_{e \in E} \left(\left(C^F_e + C^W_e \right) w_e + C^c v_e \right) + \sum_{n \in N} \left(C^O y_n + C^p u_n \right) \quad (1)$$

D. Constraints

There are five sets of constraints in this model. The first set of constraints, labelled as (2), ensures demand satisfaction and does not allow splitting demands. Constraint set (3) converts path capacity to segment capacity and segment capacity into fibers and channels. Constraint set (4) converts segment capacity to WDM units. The fourth set of constraints, labelled (5), accumulates channels on links to add the required number of ports to each node. The last set of constraints (6) converts node capacity to OXC units.

$$\sum_{p \in J_{od}} x_p^{od} = 1, \forall (o,d) \in D \quad (2)$$

$$\sum_{(o,d)\in D} R_{od} \sum_{p \in J_{od}, e \in p} x_p^{od} \leq v_e + f_e, \forall e \in E \quad (3)$$

$$v_e \leq M^W w_e + g_e, \forall e \in E \quad (4)$$

$$\sum_{e \in A_n} (v_e + f_e) \leq u_n, \forall n \in N \quad (5)$$

$$u_n \leq M^O y_n + h_n, \forall n \in N \quad (6)$$

All other decision variables are nonnegative integer.

E. The Stochastic Model

We use the general modelling framework of stochastic programming to construct a robust optimization model that will be used to get the computational results. The set of scenarios for a problem having s^* scenarios is denoted $S = 1, ..., s^*$. We use the variable z_{ods} to represent the under provisioning, for each scenario $s \in S$ and origin/destination demand $(o,d) \in D$, and P_s to denote the probabilities of the scenarios. The under provisioning value z_{ods} is the amount of demand *ods* that cannot be routed using the capacity currently installed in the network. For each value of a *penalty* parameter A, the objective to be minimized may be stated as the sum of the design cost and a penalty cost as follows:

$$\min \sum_{e\in E} 2C_e^F f_e + \sum_{e \in E} \left(\left(C_e^F + C_e^W \right) w_e + C^c v_e \right) + \sum_{n \in N} \left(C^O y_n + C^p u_n \right) + A \left(\sum_{s \in S} P_s \sum_{od \in D_s} z_{ods} \right) \quad (7)$$

Note that since the cost of the design is being minimized in the objective value we do not use a variable to represent over provisioning.

The demand constraints (2) can be modelled, using the decision variables x_p^{ods} for the routing path of the demands in each scenario, as follows:

$$\sum_{p \in J_{od}^s} x_p^{ods} R_{od}^s + z_{ods} = R_{od}^s, \quad \forall (o,d) \in D_s, s \in S \quad (8)$$

Constraint set (9), corresponding to the constraint set (3), converts path flows to segment flows and segment capacity into fibers and channels for each scenario.

$$\sum_{p \in J_{od}^s, e \in p} x_p^{ods} R_{od}^s \leq v_e + f_e, \forall e \in E, (o,d) \in D_s, s \in S \quad (9)$$

Constraint sets (4), (5) and (6) that, respectively, convert segment capacity to WDM units, accumulate channels on links to add the required number of ports to each node and convert node capacity to OXC units are not affected by the scenarios. They are now labelled (10), (11) and (12).

$$v_e \leq M^W w_e + g_e, \forall e \in E \quad (10)$$

$$\sum_{e \in A_n} (v_e + f_e) \leq u_n, \forall n \in N \quad (11)$$

$$u_n \leq M^O y_n + h_n, \forall n \in N \quad (12)$$

The last set of constraints bounds the demands.

$$0 \leq z_{ods} \leq R_{od}^s, \forall (o,d) \in D_s, s \in S \quad (13)$$

The robust optimization model of minimizing (7) subject to (8) to (13) is only one of several models that can be used to help design a network when the demand forecast is uncertain. Other possibilities include the *mean-value* model, the *worst-case* model and other *robust optimization* model, which will be used to design the network in future research. The specific choice of penalty function is problem dependent, and it also has implications for the accompanying solution algorithm. Mulvey et al. [16] recommended using some type of quadratic function as penalty function. A popular approach is the mean/variance model. However, when modern solvers such as Cplex are used to obtain near optimal solutions to real-world problem instances, it is not possible to use quadratic but linear functions to generate only linear models. This is the rationale behind the generation of a procedure based on scatter search optimization metaheuristic that can deal with any penalty function developed for the real problem we are working on.

```
procedure Scatter Search
begin
    repeat
        GeneratePopulation(InitPop);
        repeat
            GenerateReferenceSet(RefSet);
            repeat
                SelectSubset(SubSet);
                CombineMethod(SubSet, CurSol);
                ImproveMethod(CurSol, ImpSol);
                UpdateReferenceSet(RefSet,ImpSol);
            until (Stopping Criterion 1);
        until (Stopping Criterion 2);
    until (Stopping Criterion 3);
end.
```

Fig. 1. Scatter Search

IV. A Scatter Search Solution Approach

This section summarizes a scatter search (SS) [12] approach for solving the wavelength division multiplexing and optical cross-connect routing and provisioning problem under demand uncertainties. Scatter Search is a population-based metaheuristic that uses a reference set to combine its solutions and construct others. The method generates a reference set from a population of solutions. Then a subset is selected from this reference set. The selected solutions are combined to get starting solutions to run an improvement procedure. The result of the improvement can motivate the updating of the reference set of solutions.

The initial population must be a wide set of disperse solutions. However, it must also include good solutions. Several strategies can be applied to get a population with these properties. The initial population can be obtained by a procedure that provides at the same time disperse and good solutions.

A set of good representative solutions of the population is chosen to generate the reference set. The good solutions are not limited to those with the best objective values. By good representative solutions we mean solutions with the best objective values as well as disperse solutions; in the sense that they would be improved to reach different local minima by a local search. Indeed, a solution may be added to the reference set if the diversity of the set improves. So the reference set must consist of a set of disperse and good solutions selected from the populations. The criteria for updating the reference set, when necessary, must be based on comparisons and measures of diversity and quality between the new solutions and the existing solutions.

Figure 1 shows a high level pseudocode for the Scatter Search Metaheuristic. The six procedures involved in the Scatter Search are the following:

1. *The initial population generation method,* that generates the initial population $InitPop$.
2. *The reference set generation method,* which selects the set $RefSet$ that consists of the "best" solutions in the population $InitPop$.
3. *The subset generation method,* which chooses a subset $SubSet$ that consists of r solutions in the reference set to apply the next combination procedure.
4. *The solution combination method,* which is a procedure that combine the solutions in $SubSet$ to get the current solution $CurSol$.
5. *The improvement solution method.* It is the procedure to improve the current solution $CurSol$ to get a better solution $ImpSol$.
6. *The reference set updating method.* It is the procedure to decide when and how to update the reference set taking into account the state of the search.

The initial population generation method generates solutions using the metaheuristic procedure developed in [15] for the basic routing and provisioning problem. This solution procedure employs the notion of a base network, which initially consists of the current network design. A base is a network design that is not capable of handling a set of forecasted demand requirements. The initial base is the current network design, which may or may not include WDM components. Since the demands share the existing capacity of the network, the initial base may yield poor estimates of the incremental cost of routing the forecasted demand. However, as the process iterates, the base network evolves and the estimated cost of routing a demand becomes more accurate. An evolved base network includes additional equipment, which has been tentatively added to the initial base (i.e., the current network). So, when a demand is considered for routing on an evolved base network, this demand can share the additional capacity with other demand requirements, making the cost estimates more accurate, due to the decreasing fraction of the capacity that is not shared for costing purposes. A solution is a network design that is capable of handling all the demand requirements.

The solution approach builds a list of paths for each demand. The paths for a given demand are found calculating the incremental cost of routing the entire demand in the base network. For example, a possible path would be to add the necessary fiber, WDM and OXC systems to create a segment from the origin to the destination of a given demand. Other paths are created using alternative ways of carrying the demand from origin to destination, which would most likely imply adding WDMs and OXCs to some links and nodes.

For the routing and provisioning problem with uncertain demands every solution in the population is obtained by selecting a random number of demands for each scenario and routing them through the existing network design using the metaheuristic procedure described above.

Once the design variables have been fixed, a multicommodity maximum flow problem must be executed for each scenario in order to achieve the amount of demand that can not be carried using the capacity installed in the optical network. The resulting problem, that provides the under provisioning cost, is then stated as follows:

$$\min \sum_{od \in D_s} z_{ods} \qquad (14)$$

Subject to:

$$\sum_{p \in J^s_{od}} R^s_{od} x^{ods}_p + z_{ods} = R^s_{od}, \forall (o,d) \in D_s \qquad (15)$$

$$\sum_{p \in J^s_{od}, e \in p} R^s_{od} x^{ods}_p \leq v_e + f_e, \forall e \in E \qquad (16)$$

The set of constraints labelled as (15) are demand constraints for a given scenario.

Constraint set (16) converts path flows to segment flows and segment capacity into fibers and channels for a given scenario.

The right hand coefficients in constraint set (16) are the sum of two design variables, which represent the number of WDM channels and fiber pairs on segment e. Therefore, they are a constant since the design variables are fixed. We use Cplex 7.5 to solve this problem [6].

The notion of best solutions used in the *reference set generation method* is not limited to a measure given exclusively by the evaluation of the objective function. In particular, a solution may be added to the reference set if the diversity of the set improves even when the objective value of the solution is inferior to other solutions competing for admission into the reference set. The reference set that we generate consists of 10 solutions.

The combination parameters of the *solution combination method* are used to modulate the intensification and/or diversification of the search. For every segment $e \in E$, we obtain the maximum number of channels installed in the networks in $SubSet$. Then, if the installation cost of this number of channels is smaller than the penalty cost of not installing that capacity, the maximum number of channels is installed in the combined solution. Otherwise, we obtain the minimum number of channels installed in the networks in $SubSet$. Then, if the installation cost is smaller than the penalty cost, we add the minimum number of channels to the combined solution. If the decision is not to install any capacity on that segment, then the design variables v_e and f_e are equal to zero.

The *improvement solution method* applied to the current solution is based on changing one demand from its current path to another. We execute a local search procedure based on these moves to achieve the improved solution. The set of possible paths for each demand ranges from 0 up to $kmax$, which indicates the maximum number of paths calculated for each demand. If the chosen path for a demand is equal to 0, then we do not install any capacity to route that demand and, therefore, the penalty term increases. The neighborhood search examines moves employing a given ordering of the demands. That is, the first candidate move is to reassign the demand that is at the top of the list. When reassigning this demand the design variables change, but the model that has to be solved in order to achieve the penalty term for a given scenario modifies only the right hand coefficient in constraint set (16). Therefore, we have to solve the modification of the current model. To do this, we do not have to start a new model form scratch, but instead we can take the existing model and change it to our needs. This is done by calling the Cplex modification methods. When an extracted model is modified, the modification is tracked in the Cplex object. Cplex not only track all modification of the model it has extracted, but also tries to maintain as much solution information from a previous invocation of *solve()* as is possible and reasonable.

If the new solution is better than the worst in the current reference set, then the reference set is updated. If an improving move that involves reassigning the first demand in the list cannot be found, then the second demand is considered. The process continues until a demand is found for which a reassignment of paths leads to an improving move. If all the demands are examined and no improving move is found, the local search is abandoned.

V. A Test Problem

To illustrate the practical application of the robust optimization methodology for the routing and provisioning problem under demand uncertainties, a test problem has been solved, for several values of the penalty parameter, using the scatter search procedure presented in this manuscript. The metaheuristic was implemented in C and compiled with Microsoft Visual C++ 6.0. All test runs were performed on a PC with one processor at 1.0-Ghz and 256 Mbytes of RAM. The network topology for the Extant0D problem is illustrated in Figure 2 and its characteristics are given in Table I.

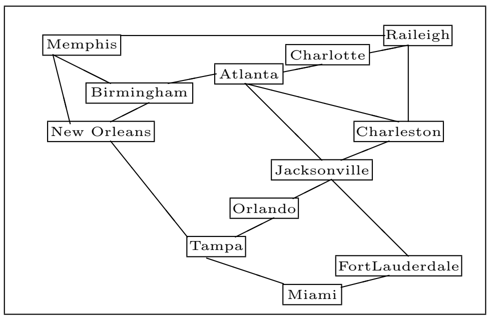

Fig. 2. Network for Extant0D Problem.

TABLE I
Characteristics of Test Problem.

Name	Extant0D
Total Nodes	12
Total Links	17
Total Demand Pairs	19
Number of Paths/Demand	5
Total Demand Scenarios	3

The program includes five candidate paths for each of the 19 demand pairs. Figure 3 shows the graph of the design cost versus the penalty cost when considering different penalties A to represent the cost of infeasibility for the test problem. Table II summarizes the design and penalty costs for every value of A.

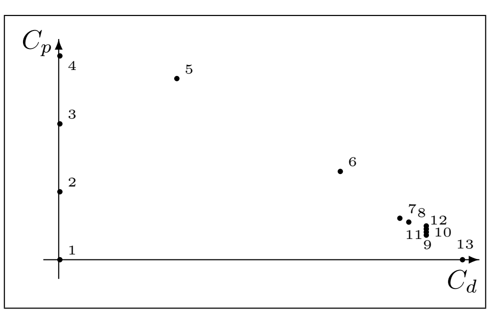

Fig. 3. Design Cost (C_d) versus Penalty Cost (C_p).

Note that if the penalty A is equal to zero, then the cost of not routing the set of demands for each scenario is equal to zero. In addition, the decision is not to increase the capacity of the network and the design cost is also equal to zero. This case is represented by 1 in Figure 3. When increasing the penalty value A the design cost increases while the under provisioning cost decreases as is shown in Figure 3 and Table II.

TABLE II
Design/Penalty Cost for Extant0D instance.

	A	Design Cost	Penalty Cost
1	0	0	0
2	50000	0	1770000
3	100000	0	3540000
4	150000	0	5310000
5	200000	3045000	4720000
6	250000	7301400	2300000
7	300000	8848800	1080000
8	350000	9082800	980000
9	400000	9538800	640000
10	450000	9538800	720000
11	500000	9538800	800000
12	550000	9538800	880000
13	600000	10482400	0

VI. Conclusions

In this paper we consider the problem of planning the capacity expansion of an optical network with WDM systems under demand uncertainty. We present a robust optimization model for the WDM and OXC routing and provisioning problem that combines the model robustness and the solution robustness. A network design gives the number and placement of fiber pairs, WDM equipments and channels, and OXC units and ports. The installation cost of the equipment gives the solution robustness of the design. The amount of demand of the scenarios that can not be served with the installed capacity gives the under provisioning. The penalty cost of the not routed demand measures the model robustness of the design. The corresponding problem is solved by a Scatter Search procedure. The procedure is useful also for other robust optimization models like the mean-value model, the worst-case model and models that use nonlinear penalty functions.

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