On the re-allocation and location of schools as required by the educational reform in the island of La Palma

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Resumen

The implantation of the new Spanish educational system creates problems concerning with the re-allocation of the existing schools, the location of new schools and the assignment of pupils. A location-allocation model is proposed for solving these problems and it is applied to the island of La Palma.

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1. Introduction.

The implantation of the new educational system in primary and secondary schools in Spain in accordance with the *Ley de Ordenación General del Sistema Educativo* (LOGSE), creates some problems that the local administration must face. In this paper, the problems related to the location of units required (classrooms), the assignment of pupils to these units and the re-allocation of the existing schools, are considered; other aspects as the adaptation of the teachers to the LOGSE and the different fields existing in certain educational levels (for example, different branches of the vocational training) are not taken into account.

In the new system, the non-university education is organized in four levels composed by sublevels or annual courses; the first level is the kindergarten (3-5 age group), the second level is the primary school (6-11 age group), the third level is the secondary school (12-15 age group) and the fourth level is the post-secondary school (16 and over age group).

The existing schools can be converted to the LOGSE system according to the following rules. These schools are classified by the administration in three types: 1, 2 and 3. In schools of type 1, only the first level can be established. In schools of type 2, the first, second and third level can be established; however, for type 2 schools, if the first or second level is established then the third one is not established. In schools of type 3, the third and fourth levels can be established.

Given the existing schools and the set of possible locations of new units of each level and the demand of pupils at every village, the local administration want to know the number of units for each course in each potential location and existing school, and the assignment of the demand to these schools satisfying the following conditions:
- the time traveled by each pupil is less than a given threshold,
- the number of units is minimum,
- the utilization of existing units is maximum,
- the total travel time is minimum.

In this paper, a location-allocation model is proposed to solve a problem raised by the implantation of the new Spanish educational system. This problem concerns with the location of the units required (classrooms), the assignment of pupils and the re-allocation of existing schools to the LOGSE system. The objectives are to minimize the number of units required, to maximize the utilization of the existing units and to minimize the total travel time. Similar problems but without the re-allocation of schools are considered in [5] and [6]. In these papers, the assignment of pupils is determined considering only the travel time and the capacity constraints. [1] and [4] consider also constraints to force the assignment of pupils from the same neighborhood or town to the same school or town. In [2] several models are given for the commonly called controlled choice (CC) to school assignment, these models allow parents to pick a school within the district as long as a racial balance is maintained and the school is not overcrowded.

Some papers consider the relations between demographic aspects and educational planning, see [3] and [8]. The influence of the demographic changes on the solution is not explicitelly considered in our research. However, if the population tends to fall, some units could become vacant soon after building them. This was the reason to consider modifications of the solution that try avoid the installation of new units and to choose to move the pupils.

The model is applied in the Canary Island of La Palma. In this application, the problem was too large to be solved by the small-size computer package available; the package admits problems of that size but in this particular case a solution was not obtained. To reduce the complexity of the problem, it is split into several smaller problems using cluster analysis techniques. A single link cluster splits the territory into zones and the problem for the levels 1 and 2 is solved for each zone. In addition to the solutions calculated for the zones, some other options obtained by changes of these solutions are proposed, these options represent alternative educational policies. The model is presented in the next section and the application to the schools in La Palma is analyzed in section 3.

2. The Model.

Let us consider the following notation:
- N = set of nodes; villages.
- L = set of levels; $L = \{1, 2, 3, 4\}$.
- K_l = set of courses of level l, $l \in L$.
- K = total set of courses; $K = \bigcup_{l \in L} K_l$.
- I_k = set of demand nodes of course k, $k \in K$.
- J_k = set of service nodes of course k, $k \in K$.
- H_j = set of existing schools in node j, $j \in N$.
- M_r = set of existing schools of type r, $r = 1, 2, 3$.
- H = total set of existing schools; $H = \bigcup_{j \in N} H_j = M_1 \cup M_2 \cup M_3$.

The following input data are available:
- δ_{ij} = distance between nodes i and j; $i, j \in N$.
- α_{ij} = the inverse of the travel speed from i to j (it depends on the kind of road).
- γ_l = threshold for travel time at level l, $l \in L$.
- ρ = upper bound on the number of pupils in a unit.
- β_{ik} = demand (number of pupils) of course k in node i, $i \in N$, $k \in K$.
- θ_h = number of units in school h, $h \in H$.

The decision variables are denoted by:
- x_{ijk} = number of pupils of course k in node i assigned to node j, $k \in K$, $i, j \in N$.
- z_{jk} = number of units of course k located in node j, $k \in K$, $j \in N$.
- w_{hk} = number of units of course k located in school h, $k \in K$, $h \in H$.

The objectives are the following:

1. Minimize the number of units,

$$\text{mín} \sum_{l \in L} \sum_{k \in K_l} \sum_{j \in N} z_{jk}. \tag{1}$$

2. Maximize the number of the utilized existing units,

$$\text{máx} \sum_{h\in H}\sum_{k\in K} w_{hk}.. \tag{2}$$

3. Minimize the total travel time,

$$\text{mín} \sum_{l\in L}\sum_{k\in K_l}\sum_{i\in N}\sum_{j\in N} \alpha_{ij}\delta_{ij}x_{ijk}. \tag{3}$$

The model includes the following constraints:

- The time travelled by every pupil at each level has to be less than a the corresponding threshold,

$$\alpha_{ij}\delta_{ij} \geq \gamma_l \Rightarrow x_{ijk} = 0 \qquad \forall k \in K_l. \tag{4}$$

- The demand has to be totally satisfied,

$$\sum_{j\in N} x_{ijk} = \beta_{ik}, \qquad \forall i \in N, \forall k \in K_l, \forall l \in L. \tag{5}$$

- The number of pupils at each course can not exceed the capacity of the units,

$$\sum_{i\in N} x_{ijk} \leq \rho z_{jk}, \qquad \forall j \in N, \forall k \in K_l, \forall l \in L. \tag{6}$$

- The number of the existing units of level l utilized in node j can not exceed the number of units located in this node,

$$\sum_{h\in H_j}\sum_{k\in K_l} w_{hk} \leq \sum_{k\in K_l} z_{jk}, \qquad \forall j \in N, \forall l \in L. \tag{7}$$

- Re-allocation rules for existing schools,

  1. $\forall h \in M_1$:

$$w_{hk} = 0, \qquad \forall k \in (K_2 \cup K_3 \cup K_4). \tag{8}$$

  2. $\forall h \in M_2$:

$$w_{hk} = 0, \qquad \forall k \in (K_3 \cup K_4). \tag{9}$$

or

$$w_{hk} = 0, \qquad \forall k \in (K_1 \cup K_2 \cup K_4). \tag{10}$$

  3. $\forall h \in M_3$:

$$w_{hk} = 0, \qquad \forall k \in (K_1 \cup K_2). \tag{11}$$

- Capacity constraints in existing schools,

$$\sum_{k\in K} w_{hk} \leq \theta_h, \qquad \forall h \in H. \tag{12}$$

- Integrality constraints.

$$x_{ijk}, z_{jk}, w_{hk} \in Z_+, \qquad \forall i, j \in N, \forall k \in K_l, \forall l \in L, \forall h \in H. \tag{13}$$

The size of the problem can be reduced by some considerations.

- For some courses the set of demand nodes $I_k \neq N$ and for higher levels the set of service nodes J_k is much smaller than N.
- For each node i and course k, let $S_{ik} = \{j \in N/\delta_{ij} \leq \gamma_l, k \in K_l\}$. Then we only need to consider variables x_{ijk} for $j \in S_{ik}$ being S_{ik} usually much smaller than N and threshold times constraints are avoided.
- To take into account the possible levels in the existing schools, let

  $$T = (M_1 \times K_1) \cup (M_2 \times (K_1 \cup K_2 \cup K_3)) \cup (M_3 \times (K_3 \cup K_4)).$$

  Then we only need variables w_{hk} for $(h, k) \in T$.
- In order to control the LOGSE-type of converted schools of type 2 let variables u_h and v_h (for $h \in M_2$) be defined by:
  - $u_h = 1$ if in school h are established units of levels 1 and/or 2 and $u_h = 0$ otherwise;
  - $v_h = 1$ if in school h are established units of level 3 and $v_h = 0$ otherwise. So the number of constraints for the re-allocation rules is reduced.
- For low levels, most of pupils are assigned to schools in their nodes then the number of variables x_{ijk} can be reduced by using the relation

  $$x_{iik} = \beta_{ik} - \sum_{j \in S_{ik} - \{i\}} x_{ijk},$$

  removing the variables x_{iik} which usually have high values.

  Therefore the demand satisfaction constraints can be expressed as follows:

  $$\sum_{j \in S_{ik} - \{i\}} x_{ijk} \leq \beta_{ik}, \qquad \forall i \in I_k \cap J_k, \forall l \in L, \forall k \in K_l$$

  $$\sum_{j \in S_{ik} - \{i\}} x_{ijk} = \beta_{ik}, \qquad \forall i \in I_k - J_k, \forall l \in L, \forall k \in K_l.$$

  And the capacity of units constraints can be replaced by

  $$\sum_{i \in I_k} \sum_{j \in S_{ik} - \{i\}} x_{ijk} - \sum_{r \in S_{jk} - \{j\}} x_{jrk} + \beta_{jk} \leq \rho z_{jk}, \qquad \forall j \in I_k \cap J_k, \forall k \in K$$

  $$\sum_{i \in I_k} \sum_{j \in S_{ik}} x_{ijk} \leq \rho z_{jk}, \qquad \forall j \in J_k - I_k, \forall k \in K.$$

So the problem formulation is reduced to:

$$\text{mín} \sum_{l\in L} \sum_{k\in K_l} \sum_{j\in J_k} z_{jk}. \tag{14}$$

$$\text{máx} \sum_{(h,k)\in T} w_{hk}. \tag{15}$$

$$\text{mín} \sum_{l\in L} \sum_{k\in K_l} \sum_{i\in I_k} \sum_{j\in S_{ik}\cap J_k} \alpha_{ij}\delta_{ij}x_{ijk}. \tag{16}$$

subject to:

$$\sum_{j\in S_{ik}-\{i\}} x_{ijk} \leq \beta_{ik}, \qquad \forall i \in I_k \cap J_k, \forall k \in K_l, \forall l \in L. \tag{17}$$

$$\sum_{j\in S_{ik}-\{i\}} x_{ijk} \leq \beta_{ik}, \qquad \forall i \in I_k - J_k, \forall k \in K_l, \forall l \in L. \tag{18}$$

$$\sum_{i\in I_k\cap S_{ik}} x_{ijk} \leq \rho z_{jk}, \qquad \forall j \in J_k, \forall k \in K_l, \forall l \in L. \tag{19}$$

$$\sum_{k\in K_l} \sum_{(h,k)\in T} w_{hk} \leq \sum_{k\in K_l} z_{jk}, \qquad \forall j \in J_k, \forall l \in L. \tag{20}$$

$$\sum_{(h,k)\in T} w_{hk} \leq \theta_h, \qquad \forall h \notin M_2. \tag{21}$$

$$\sum_{k\in K_1\cup K_2} w_{hk} \leq \theta_h u_h, \qquad \forall h \in M_2. \tag{22}$$

$$\sum_{k\in K_3} w_{hk} \leq \theta_h v_h, \qquad \forall h \in M_2. \tag{23}$$

$$u_h + v_h \leq 1, \qquad \forall h \in M_2. \tag{24}$$

$$x_{ijk} \in Z_+, \qquad \forall i \in I_k, \forall j \in S_{ik} - \{i\}, \forall k \in K_l, \forall l \in L. \tag{25}$$

$$z_{jk} \in Z_+, \qquad \forall j \in J_k, \forall k \in K_l, \forall l \in L. \tag{26}$$

$$w_{hk} \in Z_+, \qquad \forall (h,k) \in T. \tag{27}$$

$$u_h, v_h \in Z_+, \qquad \forall h \in M_2. \tag{28}$$

The reduced problem has

$$\sum_{k\in K} \left(\sum_{i\in I_k} |S_{ik}| - |I_k| \right) + \sum_{k\in K} |J_k| + |T| + 2|M_2|$$

variables and

$$\sum_{k\in K} |I_k| + \sum_{k\in K} |J_k| + \sum_{l\in L} |\bigcup_{k\in K_l} J_k| + |H| + 2|M_2|$$

non-integrality constraints, where $|S|$ denotes the cardinal of the set S.

3. Application.

The model introduced in section 2 is applied in the island of La Palma which is a mountainous island of the Canary Archipelago. This island has a population of 78867 (according to the last census) and an area of 706.2 km^2. Some of the villages are isolated and sparsely populated, and the roads between some zones are not good. There are 92 public schools, 2 of type 1, 82 of type 2 and 7 of type 3. There are also two special cases not mentioned in the description of the model but must be took into account in the solution. First, there is one semi-private school where all levels can be established. Second, in some very small villages there is a special type of school, the “unitarian”school, where pupils of different courses, even of different levels, are in the same unit receiving class simultaneously (a teacher teaches pupils of different courses in the same unit at the same time). This is because in those cases the sets S_{ik} are empty (even enlarging the thresholds) and the demands β_{ik} are very low but not null.

There are 130 demand nodes for the four levels. The potential locations for units of levels 1 and 2 are the same 130 nodes. For these levels a threshold of 4.2 minutes is first considered. For levels 3 and 4, the units will be located only in 7 nodes of the island to be determined by the administration. In these cases the problem is the assignment of pupils to these nodes and the calculation of the units required.

To express the total travel distances, that is the third objective of the model, in terms of travel times, the roads are classified in three types corresponding to average speeds of 50 km/h, 20 km/h and 10 km/h, respectively. The worst roads are paths and non-asphalted roads, in some cases only special vehicles can run along these roads.

The threshold of 4.2 minutes corresponds to 3.5 km on the best roads. For this threshold, the reduced problem has less than 9746 variables and 3172 constraints. In any case the problem was too large to be solved with the available optimization package (LINDO Industrial) and it was split considering first the problem for levels 1 and 2, and afterwards the problem for levels 3 and 4. The problem for levels 1 and 2 was still too large and it was again split into smaller problems by consideration of the thresholds and a single linkage cluster. The cluster splits the territory into zones in such a way that the travel time between nodes in different zones is greater than the threshold. For 4.2 minutes, 59 zones are obtained, 49 with one village and the biggest one with 21 villages. For a greater threshold a smaller number of zones can be obtained, so for 9 minutes, 20 zones are obtained, 15 with one village and the biggest one with 51 villages. There is a problem associated to each zone; for the biggest zones a new split can be made if necessary considering the levels individually or in groups.

For levels 1 and 2, the multiobjective problem is solved by using a preemptive priority method (see [7]) taking into account the objectives ordered as shown in the formulation of the model. The locations for units of level 3 and 4 are fixed and the pupils are assigned to the nearest node where units of these levels can be installed, the number of units required is calculated and the schools for levels 3 and 4 are determined considering the solution obtained for levels 1 and 2.

The original solution was modified according to some criteria in order to give different options to the administration. These alternative solutions can violate some constraints of the model but they would be better from certain points of view of the educational administration. Aspects as the following were considered:

- The capacity of the units is rather elastic, people who determined the assignment manually could slightly modify capacities if necessary, thus violating capacity constraints. If too few pupils were assigned to a particular unit then the pupils of this unit could be assigned into other units, thus reducing the number of units required.
- In some nodes, units of nonconsecutive courses are located; in this case, the administration can decide to use these units for consecutive courses, for example for courses corresponding to the lowest school age, or to establish a system of "unitarian"school, trying to minimize the travel time for the youngest pupils.
- In some zones, the minimization of the total number of units forces the locating of new units in certain nodes while there are units in other nodes of the same zone that are not used, this would require an investment that can be avoided with a better utilization of resources although it is at the expense of an increasing of the total number of units (other efficient solutions can also be calculated) or a violation of the thresholds.

In this paper, the results for an area of 5 zones including 17 villages belonging to the municipalities of *Santa Cruz* (the capital of the island) and *Las Breñas* that are situated in the eastern part of the island are shown. The nodes in this area and the transportation network connecting them are represented in figure 1. The numeration of nodes and travel times in minutes between nodes are included. The nodes that are not in the 5 zones but connected to it by an arc are represented by a empty circle; i.e. nodes numbered 93, 13, 59, 126 and 121.

The results for levels 1 and 2, are summarized in tables 1 to 3. Pupils of level 1 can be in the same classroom ($|K_1| = 1$) but level 2 includes 6 courses; $K_2 = \{6, 7, 8, 9, 10, 11\}$. In these tables, for each zone and each node, the demand for each course and different solutions are shown. When there not enough existing units in a node, existing units and new units are separated by a dash. In every table, option 1 is the solution obtained applying the preemptive priority method considering that the first objective is the minimization of the units. The other options shown are "corrections.ºf option 1 according to some aspects mentioned above.

Table 1 shows the result for zones 2 and 3 where option 1 requires 14 new units. For options 2 and 3 the number of new units is reduced by assigning the pupils of zone 3 to units in zone 2 but the threshold is violated. For option 2 new units are not required and the maximum travel time is 19.5 minutes Option 3 requires 3 new units located in node 17 and the maximum travel time is 10.5 minutes. Note that the node where the location of new units for all levels requires the minimum number of them is node 17. Table 2 shows the utilization of existing schools for the different solutions in zone 2.

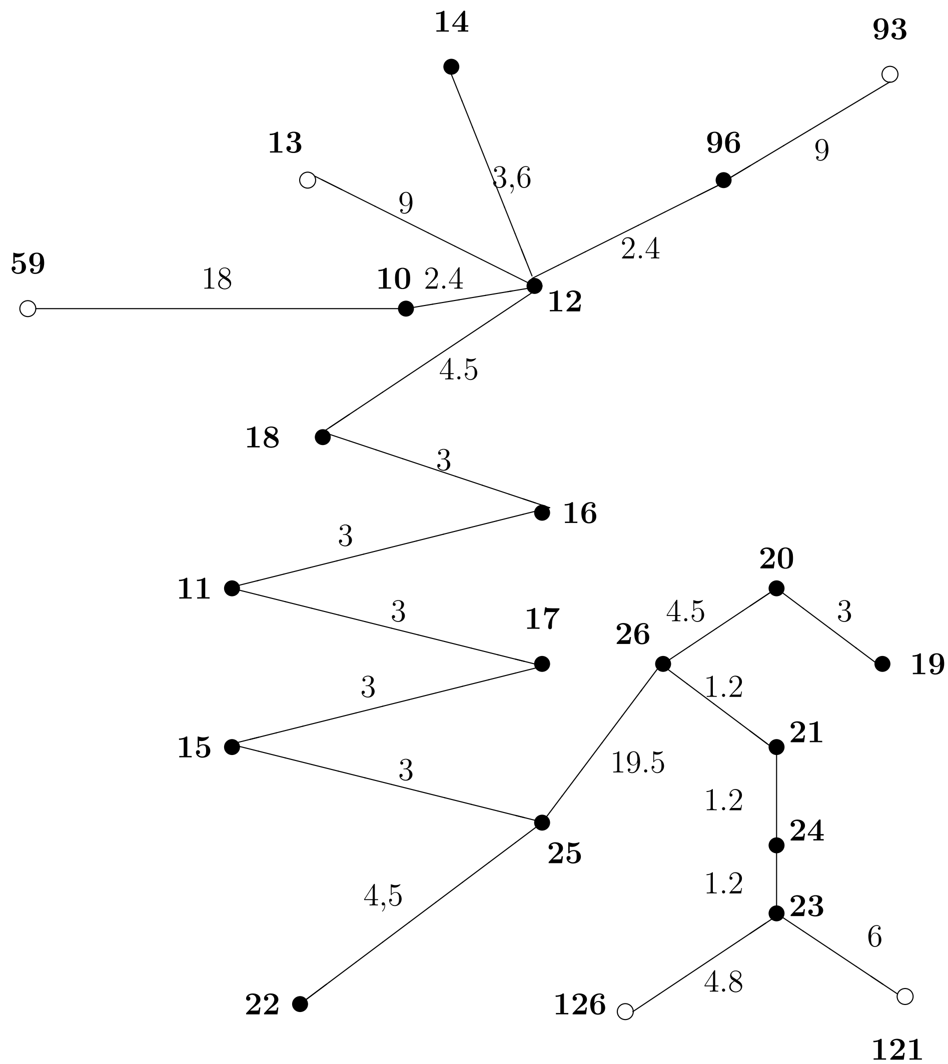

Figura 1: Transportation network in the eastern area of La Palma. Node numbers are in boldface letter and travel times in minutes.

Node		total	l=1	l=2	k=6	k=7	k=8	k=9	k=10	k=11
j=11	pupils	66	20	46	8	5	7	7	8	11
op.1	units	3-1	1	2-1	1	0	0	0	1	0-1
op.2-3	units	3	1	2	1	1	0	0	0	0
j=15	pupils	35	8	27	3	4	3	7	3	7
op.1	units	1	0	1	0	0	0	0-1	0	0
op.2-3	units	0	0	0	0	0	0	0	0	0
j=16	pupils	52	10	42	9	4	4	6	8	11
op.1-3	units	0	0	0	0	0	0	0	0	0
j=17	pupils	77	19	58	7	11	11	10	11	8
op.1	units	4	1	3	0	1	1	1	0	0
op.2	units	4	1	3	1	1	1	0	0	0
op.3	units	4-3	1	3-3	1	1	1	0-1	0-1	0-1
j=18	pupils	274	93	181	23	29	38	30	31	30
op.1	units	15	4	11	1	2	2	2	2	2
op.2	units	20	4	16	1	2	3	3	3	4
op.3	units	17	4	13	1	2	3	2	2	3
j=25	pupils	96	33	63	12	14	12	6	10	9
op.1	units	2-5	2	0-5	0-1	0-1	0-1	0	0-1	0-1
op.2-3	units	2	2	0	0	0	0	0	0	0
zone 2	pupils	600	183	417	62	67	75	66	71	76
op.1	units	24-7	8	16-7	2-1	3-1	3-1	3-1	3-1	2-2
op.2	units	29	8	21	3	4	4	3	3	4
op.3	units	26-3	8	18-3	3	4	4	2-1	2-1	3-1
j=22	pupils	33	11	22	3	3	4	5	3	4
op.1	units	0-7	0-1	0-6	0-1	0-1	0-1	0-1	0-1	0-1
op.2-3	units	0	0	0	0	0	0	0	0	0
zone 3	pupils	33	11	22	3	3	4	5	3	4
op.1	units	0-7	0-1	0-6	0-1	0-1	0-1	0-1	0-1	0-1
op.2-3	units	0	0	0	0	0	0	0	0	0

Table 1: Zones 2 and 3. Number of pupils and units required for levels 1 and 2.

School	Node	Type	Units	Units utilized				
				l=1	l=2	l=3	l=4	Total
37	11	2	3	1	2	0	0	3
38	18	2	20					
Option 1				4	11	0	0	15
Option 2				4	16	0	0	20
Option 3				4	13	0	0	17
40	17	2	4	1	3	0	0	4
41	25	2	2	2	0	0	0	2
TOTAL			29					
Option 1				8	16	0	0	24
Option 2				8	21	0	0	29
Option 3				8	18	0	0	26

Table 2: Utilization of existing schools in zone 2 for different options.

For levels 1, 2 and 3, the demand is the number of people in the corresponding age group. However for level 4 this does not happen then we consider the results for 3 different ways of estimating the demand. In the first case the demand is the 16-17 age group, in the second case we add a percentage of the 18-20 age group and in the third case we add a percentage of 16-20 age group. These percentages are based on the number of pupils of these ages in level 4 in the last years. The units required for levels 3 and 4 in a wider area of the whole eastern part of the island between the municipalities of *Barlovento* and *Fuencaliente* are shown in table 3.

Node	Level l=3				Total	Level l=4		Total	TOTAL
	1	2	3	4		1	2		
24	4	4	4	4	16	4	4	8	24
						4	6	10	26
						2	4	6	22
87	7	6	7	6	26	8	7	15	41
						8	16	24	50
						5	13	18	44
96	18	16	16	17	67	19	18	37	104
						19	46	65	132
						17	42	59	126
125	3	3	3	4	13	4	3	7	20
						4	5	9	22
						1	3	4	17

Table 3:Units required for levels 3 and 4 in the eastern part of the island for 3 different methods of estimating the demand.

4. Concluding Remarks.

This research is related with a project supported by the *Consejería de Economía y Hacienda* of the Canary Government. At the moment of the last revision of this paper, the authorities are with the conclusion at hand. The final decision about the re-allocations is not made. There is a social movement in all the country againts the distance that have to be travelled by pupils in rural zones. A previous study was used to evaluate the possible consequences of the re-allocation criteria.

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