

# Capacity Expansion of Fiber Optic Networks with WDM Systems: Problem Formulation and Comparative Analysis

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**Abstract.** In this paper we provide an arc-path formulation for the problem of locating and sizing Wavelength Division Multiplexing (WDM) and Optical Cross-Connect (OXC) equipment in a fiber optic network. The problem is heuristically solved by limiting the number of paths for each demand and applying CPLEX to the resulting formulation. The results obtained are then compared to those given by a metaheuristic method previously developed for this problem. A set of real and randomly generated instances with different characteristics associated with actual telecommunication networks are used to assess the performance of both solution approaches.

**Keywords:** DWDM, Provisioning, Routing, Mixed Integer Formulation, Scatter Search.

## 1 Introduction

The problem that we address in this paper results from the need to expand capacity of telecommunication networks built with fiber optics technology. A set of forecasted demands trigger the planning of expanding capacity of a fiber optics network that is not capable of handling the estimated future demands. The typical solution to this problem consists of determining where and how much fiber to add to the network. However, recent technology, such as Wavelength Division Multiplexing (WDM) and Optical Cross-Connect (OXC) equipment, gives telecommunication companies the option of expanding capacity without necessarily laying additional fiber. Hence, when considering these new technologies, the problem is to decide whether to add fiber or WDM and OXC equipment to expand the capacity of current fiber.

Wavelength Division Multiplexing is the transmission of multiple laser signals at different wavelengths (colors) in the same direction, at the same time and over the same strand of fiber [9]. WDM with more than eight frequencies, called Dense

Wavelength Division Multiplexing (DWDM) creates multiple bi-directional 'virtual fibers' per physical fiber. DWDM solves the bandwidth bottleneck resulting from growth in data traffic, because it is an emerging technology that increases transportation capacity while preserving optical fiber equipments previously installed. Hence, DWDM provides carriers the flexibility and scalability they need to deploy capacity when and where it is needed.

DWDM systems are equipped with amplifiers that allow transmission in one channel. However, most of the capacity cost is related to the channel cards. Channel cards are added as needed and their cost is charged to the design accordingly. This means that a system capable of handling up to 96 channels can be installed where only eight channels are active, and the design would only consider the cost of equipping the eight active channels. To use WDM technology, one multiplexer must be placed at each endpoint of the fiber link. For each wavelength, or channel in use, a card must be also placed at each endpoint of a network link. Each WDM channel is bi-directional and has the same capacity as a pair of fibers.

Amplification is the process restoring the optical signal to its original optical power and without distortion after the signal has lost power when passing through a strand of fiber. This process is particularly important in DWDM environments, where the same strand of fiber is used to send multiple laser signals at different wavelengths. Most amplifiers do not have electronic elements, that is, they are completely optical. Consequently, they do not require the classical electrical-optical and optical-electrical conversions, thereby avoiding the associated need for additional bandwidth.

WDM/DWDM technology can greatly reduce the cost of adding capacity in long-haul carrier networks. This technology is without a doubt useful, as it would cost an insane amount of money to lay new fiber to meet constantly increasing demand. The first systems in the market were optimized for long-haul inter-exchange applications. Competitive inter-exchange carriers, such as Qwest and IXC Communications, have already created new markets for optical capacity by leasing specific wavelengths to other carriers. The WDM/DWDM equipment deployed in North America has already helped to relieve long-haul network congestion experienced in 1997 and 1998 as Internet and data traffic put unprecedented demands on the existing capacity. Perhaps more importantly, WDM is enabling a new "optical layer" of telecommunication networks, in which traffic flows can be aggregated and routed more efficiently (by suitable wavelength grooming) and restored quickly and reliably after network failures [5].

While long-haul carriers understood the economic advantages of DWDM since the inception of this technology, local access companies did not immediately perceive an advantage of deploying WDM/DWDM systems when compared to adding more fiber. Recently, however, some local access companies are starting to investigate the use of WDM/DWDM systems as an attractive alternative to adding fiber to metro optical networks. The local network is challenging for carriers because of the rapid growth in demand and changes in traffic patterns. The consequence of this is that access networks must support many different service

interfaces and at the same time, it must be scalable, reliable and able to allocate bandwidth on demand. The metropolitan interoffice network is not less dynamic than the access network, because the changes in the access network directly impact the interoffice network. Therefore, rapid response to changes in the access network is critical issue for the telecommunications industry. Another factor that makes WDM/DWDM technology attractive is that vendors have been able to reduce costs to a point where its application in short-haul (metropolitan area and local access) networks is cost-effective.

Before describing the optimization problem and presenting the corresponding mathematical formulation, we provide a brief description of the main components of the WDM/DWDM technology. OXCs are small space-division switches without waveband selectivity that can switch an optical signal from one wavelength to another on multi-fiber WDM/DWDM systems or on a single fiber. They route the signals on each input port to one or more selected output ports. Providers offer several OXC sizes, such as 1616 (i.e., 16 incoming wavelengths that can be fully switched among 16 outgoing wavelengths) and up to 6464 or more, with 512512, 10241024 and even larger sizes contemplated for future product offerings.

Each WDM/DWDM system must originate and terminate at an OXC or DCS (Digital Cross-Connect Signal) port. Also, an OXC or DCS port is needed to add or drop traffic at the origin and destination of each demand carried by the network. The OXC and DCS ports are bi-directional. The installed base of DCS machines generally lacks OC-48 ports, so that each DCS OC-48 port may physically consist of 48 DS3 ports. Since we will be modelling at the OC-48 level, we can consider OXC and DCS equipment to be functionally equivalent.

Optical signals originate and terminate at network nodes, which are typically SONET (Synchronous Optical Network) [9] ring nodes carrying traffic expressed in OC-48 units. (OC-3 and OC-12 rings have been used in the past and OC-192 rings will be used in the future.)

Formal mathematical modelling and optimization techniques can reduce the costs of deploying WDM systems and hence speed the introduction of new optical capacity and services in both short-haul and long-haul fiber networks. Section 2 describes the capacity expansion problem using WDM systems. Section 3 formulates the problem as a mixed integer program. Section 4 briefly describes a metaheuristic procedure developed in [8], which we use for comparison purposes. Section 5 summarizes a comparative analysis between the metaheuristic and the solutions obtained by solving the mixed integer programming problems with CPLEX.

## 2 Problem Description

As indicated above, WDM/DWDM technology and its related equipment have several advantages when considering the increase of capacity of an existing SONET fiber network. We highlight these advantages in our problem description. In order to increase the capacity of a network at a minimum cost, it is necessary to decide:

1. Where to place WDM and OXC systems; and
2. How to route the traffic within the resulting network.

We assume that a network design exists and that our problem consists of adding capacity to the current network in order to carry a set of demands while satisfying technology constraints. In this paper we do not tackle the problem of restoring the network after a link failure. However, since the restoration problem can also be treated as a provisioning and routing problem, the proposed formulation for the service network is essentially the same for the restoration network, which is typically solved after the service network has been configured.

The optimization problem deals with a set of demands that the existing optical network is incapable of absorbing. Associated with each demand is an origin node  $s$ , a destination node  $t$ , and a bandwidth requirement  $d$ , expressed in OC-48 units. Optical fiber joining pairs of nodes is used to route demands through the network. Each demand can be routed either entirely on one or more bare fibers, over one or more channels of a WDM system or it can be switched from a WDM to another through OXCs. The goal of the network planner is to minimize the total cost, that is, the cost of additional fiber, WDM systems and OXC equipment.

In our context, the physical network design (i.e., the set of existing links) is given, which limits where additional optical fibers and WDM systems can be placed. The notion of a segment is used to represent a direct connection between an origin and a destination using glass-through nodes. Glass-through nodes for a segment do not use OXC systems and therefore traffic cannot be added or dropped between the origin and the destination. Each OC-48 unit uses two bare fibers or a channel of a WDM system. For convenience, we refer to the capacity required for an OC-48 unit as a channel, regardless of whether a pair of fibers or a channel of a WDM system is actually used. All links within a segment must be capable of carrying the same amount of traffic in order to satisfy the demand from its origin to its destination. In an optimal network design, each segment follows a least-cost path (with respect to fiber cost) from origin to destination. Since the shortest path from any node to any other is treated as a potential segment, the network of segments results in a complete graph, which is intractable in most practical cases. Therefore, as done in [8], we generate a subset of promising segments to be included in our problem formulation.

Once an OXC is reached, wavelengths and fibers can be rearranged. Therefore, the capacity constraints on each segment are simply that enough fiber and WDM equipment must be available on the segment to handle the number of OC-48 units assigned to it. Each individual link must have enough channel capacity to cover all demands routed over segments that uses it.

Cox et al. [1] proposed the planning problem that simultaneously addresses the provisioning, routing and survivability problems. The problem was approached using a genetic algorithm (GA) based on incrementally adding equipment to minimize the cost of routing each demand and has been compared with a meta-heuristic procedure that we have proposed in [8]. Kennington, et al. [4] formulate and solve a similar problem. In their work, however, they do not consider OXC

equipment or glass-through nodes, and therefore they don't use the concept of segments.

### 3 Problem Formulation

In this section, we present a mixed integer programming (MIP) formulation of the provisioning problem. We use an arc-path model that determines the equipment required to route a set of point-to-point demands through the network. The proposed formulation has significantly less integer variables than the segment-based formulation presented in [1] and modified in [8]. The segment-based formulation in those articles was mainly used as a mathematical definition of the problem instead of a mechanism for solving it. The formulation presented here is intended for finding optimal or near-optimal solutions to the capacity expansion problem on hand. Our formulation uses the following definitions.

#### 3.1 Data

The network topology is represented as a graph  $G = (N, E)$ , where  $N$  denotes the set of nodes and  $E \subseteq N \times N$  denotes the set of segments. Therefore, in our formulation, links and segments are equivalent in that they represent a direct connection between two points. The cost of using an individual link or a segment is correctly computed in the objective function. For each  $n \in N$ ,  $A_n$  denotes the set of segments adjacent to node  $n$ . The origin/destination node pairs corresponding to the point-to-point demands are given by  $D \subseteq N \times N$ . For each  $(s, t) \in D$ ,  $d_{st}$  denotes the bandwidth requirements in OC-48 units and  $P_{st}$  denotes the set of possible paths from  $s$  to  $t$  that can be used to route this demand. Since the set of paths used for each demand may not consist of all the possible paths from  $s$  to  $t$ , the formulation described in this section may be used as a heuristic model for the provisioning and routing problem.

#### Cost Input Data

$C_e^F$  = cost of a fiber on segment  $e$  (sum of costs per link along that segment).

$C_e^W$  = cost of a WDM unit on segment  $e$ .

$C^O$  = cost of an OXC unit.

$C^c$  = channel cost of a WDM unit.

$C^p$  = port cost of an OXC unit.

#### Capacity Data

$M^W$  = capacity of a WDM unit.

$M^O$  = capacity of an OXC unit.

#### Existing Infrastructure

$g_e$  = spare WDM channels on segment  $e$ .

$h_n$  = spare OXC ports at node  $n$ .

### 3.2 Decision Variables

- $x_p^{st} = 1$  if demand  $(s, t)$  is routed on path  $p$  and 0 otherwise.
- $f_e$  = number of stand-alone (no WDM) fiber pairs on segment  $e$ .
- $w_e$  = number of WDM units on segment  $e$ .
- $v_e$  = number of channels in the WDM systems installed on segment  $e$ .
- $y_n$  = number of OXC units installed at node  $n$ .
- $u_n$  = number of ports in the OXC systems installed at node  $n$ .

### 3.3 Objective Value

The following objective function minimizes the sum of stand-alone fiber costs (first term), WDM costs (second term) and the OXC costs (third term), when only one type of WDM and OXC systems is used.

$$\min \sum_{e \in E} 2C_e^F f_e + \sum_{e \in E} ((C_e^F + C_e^W) w_e + C^c v_e) \sum_{n \in N} (C y_n + C^p u_n) \quad (1)$$

### 3.4 Constraints

The following constraints express the requirements that all demand must be carried, that no link should be assigned more demand than its capacity allows it to carry and that no switching element should be assigned more traffic than its capacity allows.

$$\sum_{p \in P_{st}} x_p^{st} = 1, \forall (s, t) \in D \quad (2)$$

$$\sum_{(s,t) \in D} d_{st} \sum_{p \in P_{st}} \sum_{e \in p} x_p^{st} \leq v_e + f_e, \forall e \in E \quad (3)$$

$$v_e \leq M^W w_e + g_e, \forall e \in E \quad (4)$$

$$\sum_{e \in A_n} (v_e + f_e), \forall n \in N \quad (5)$$

$$u_n \leq M^O y_n + h_n, \forall n \in N \quad (6)$$

All other decision variables are nonnegative integer.

There are five sets of constraints in this model. The first set of constraints, labelled as (2), ensures demand satisfaction and does not allow splitting demands. Constraint set (3) converts path capacity to segment capacity and segment capacity into fibers and channels. Constraint set (4) converts segment capacity to WDM units. The fourth set of constraints, labelled (5), accumulates channels on links to add the required number of ports to each node. The last set of constraints (6) converts node capacity to OXC units.

## 4 Metaheuristic Solution Approach

This section summarizes a metaheuristic procedure developed in [8] that searches for optimal solutions to the provisioning and routing problem in fiber optics networks with WDM systems. This solution procedure employs the notion of a base network, which initially consists of the current network design. A base is network design that is not capable of handling a set of forecasted demand requirements. The initial base is the current network design, which may or may not include WDM components. Since the demands share the existing capacity of the network, the initial base may yield poor estimates of the incremental cost of routing the forecasted demand. However, as the process iterates, the base network evolves and the estimated cost of routing a demand becomes more accurate. An evolved base network includes additional equipment, which has been tentatively added to the initial base (i.e., the current network). So, when a demand is considered for routing on an evolved base network, this demand can share the additional capacity with other demand requirements, making the cost estimates more accurate, due to the decreasing fraction of the capacity that is not shared for costing purposes. The evolution of the base network is linked to an adaptive memory mechanism that keeps track of where new equipment is added in the best solutions recorded during the search. A solution is a network design that is capable of handling all the demand requirements.

The solution approach builds a list of paths for each demand by making use of an efficient implementation of the k-shortest path algorithm that allows the identification of a controlled set of feasible paths for each demand and that is a variant of the k-shortest path algorithm reported by Lawler in [7]. The paths for a given demand are found calculating the incremental cost of routing the entire demand in the base network. For example, a possible path would be to add the necessary fiber, WDM and OXC systems to create a segment from the origin to the destination of a given demand. Other paths are created using alternative ways of carrying the demand from origin to destination, which would most likely imply adding WDMs and OXCs to some links and nodes.

Four basic elements are common to heuristic search procedures, regardless of the specific methodology applied or strategic design choices: (1) a solution representation, (2) an objective, (3) an evaluation function, and (4) a move mechanism. The specifications for the search procedure proposed in [8] are:

**Solution representation** The construction of a solution starts with the selection of a path for each demand requirement. Once each demand is assigned to a path, the cost of the resulting design is calculated. The cost is associated with the equipment that is required to satisfy the demands using the chosen paths. A solution is fully determined by a data structure that stores the path assignments and the equipment required in each element of the original network.

**Objective** The goal of the capacity expansion problem on hand is to minimize the sum of additional fiber cost, WDM equipment cost and its terminal equipment (OXC units) cost, subject to the appropriate technology constraints.

**Evaluation** Once each demand has been assigned to a path in its list of potential paths, the evaluation of the solution consists of calculating the increase of capacity required in the elements of the network that route the demands through the assigned paths. The increased capacity is then translated into cost of installing additional fiber and adding WDMs and OXCs.

**Move mechanism** Every solution has a neighborhood, which consists of all the feasible solutions that are reached by changing a demand from one path to another.

The overall solution strategy consists of an adaptive metaheuristic method that combines ideas from scatter search [6], multi-start [3], and tabu search [2]. The procedure starts with the generation of a set of promising segments using the shortest path algorithm (with distances as weights). Segments corresponding to any existing WDM systems are also included in the promising set. The procedure uses these segments to execute the k-shortest path algorithm for each demand (with incremental costs from a base network as weights). After the execution of this step each demand has a set of paths that are used as the basis for building solutions. Given the network of segments, the spare capacity on the segments and nodes is determined. Obtaining spare capacities allows the procedure to assess incremental costs of routing demands in each segment. A detailed description of the procedure can be found in [8].

In order to compare the performance of our metaheuristic procedure with the Genetic Algorithm previously proposed by Cox et al. [1] to solve the problem, we have implemented a permutation-based algorithm that follows the same structure as the one proposed. The comparative analysis between both methods can be found in [8].

## 5 Computational Results

The goal of our experimental testing is to compare the solutions obtained by solving the capacity expansion problem on WDM networks with the metaheuristic approach developed in [8] and the formulation presented in section 3. Even if the MIP model in section 3 is solved to optimality, the solution is not guaranteed to be optimal for the original problem because the model includes only a subset of all possible segments and a subset of all possible paths that can be found with the given segments. The metaheuristic was previously tested in [8] on a set of instances based on real-world data shared by Dr. Leonard Lu of AT& T Labs. The test consisted of comparing the scatter search implementation developed in [8] with a permutation-based procedure similar to one proposed in [1]. In this paper, the set of instances are both real and randomly generated. The artificial instances use the same number of nodes as in the networks of the real instances but the demands and the exiting equipment are randomly generated. The intent in generating random instances is to analyze the performance of both methods on instances with various characteristics. We consider four different numbers of nodes and for each we generate networks with different densities according to the

number of segments. Then, for each network, several sets of uniform and clustered sets of demands are randomly created. Uniform demands are generated by randomly selecting an origin and a destination, where each pair has the same probability of being selected. Clustered demands are generated selecting a subset of nodes as "high traffic" locations and then generating a demand pattern that clusters around those nodes. For each set of instances, we show the number of nodes  $|N|$ , segments  $|E|$ , and demands  $|D|$ . For both the metaheuristic method and the mathematical model we have considered the same number of paths for each demand pair to make the results comparable. In addition, demands are not split in both cases.

The metaheuristic was implemented in C and compiled with Microsoft Visual C++ 6.0. The MIP formulation was solved with Cplex 7.5. All experiments were performed on a PC (with one processor at 1.0-Ghz and 256 Mbytes of RAM).

Table 1 summarizes the computational results for three networks with 11 nodes. The first network is 30% dense, that is, it consists of 30% of the links in the completely dense network. The second and third networks are 50% and 76% dense, respectively. For each network we have created four random sets of demands to simulate possible situations on a telecommunications network. The first set of demands, which consists of 54 demands, consists of uniformly distributed requirements. The other three sets of demands are generated in clusters, where either only a few nodes generate demand requirements or there are a few "high traffic" nodes with demands to other nodes in the network. The instance consisting of 11 nodes, 42 segments and 18 demands corresponds to a real instance.

Table 2 shows the results for three networks with 12 nodes. These networks are 30%, 50%, and 70% dense, respectively. For each network we have also created four random sets of demands. The set with 66 demands consists of uniformly distributed requirements. The other three sets have clustered demands. The instance consisting of 12 nodes, 17 segments and 19 demands corresponds to a real instance.

Columns 2, 3, and 4, contain the number of nodes, number of segments, and number of demands in each instance. Columns 5 and 6 in table 1 and table 2 show the total costs and the CPU time (in seconds) corresponding to the metaheuristic procedure and Cplex, respectively. All instances in these two tables were solved employing a set of 10 paths for each demand. The results obtained with Cplex have an optimality gap of 0.0001, which is the default value in this optimizer. Last column in both tables shows the deviation between the Cplex solution and the metaheuristic solution. In the worst case, the deviation is no greater than 0.95% in table 1 and 2.62% in table 2. For almost every instance in tables 1 and 2, the proposed metaheuristic procedure is able to reach the same solution obtained with Cplex. Computational times generally favor Cplex in these relatively small networks. Table 2. Comparative results for networks with 12 nodes Set

Table 3 shows the results obtained for two networks with 17 nodes. These networks are 19% and 50% dense, respectively. As in the case of tables 1 and 2, four sets of demands are randomly generated for each network to provide

**Table 1.** Comparative results for networks with 11 nodes

Set Name	$ N $	$ E $	$ D $	<i>Metaheur</i> Cost	<i>Metaheur</i> Time	<i>Cplex</i> Cost	<i>Cplex</i> Time	$\frac{C_M - C_c}{C_c}$ (%)	
MetroD	11	16	54	14.12	2.01	14.03	1.25	0.64	
			10	4.09	0.13	4.09	0.22	0	
			20	4.38	0.3	4.38	0.21	0	
	27	16	30	8.42	0.19	8.42	0.18	0	
			54	11.22	10.76	11.16	2.87	0.53	
			10	2.75	0.13	2.75	0.13	0	
	42	16	20	4.26	0.25	4.26	0.29	0	
			30	6.46	0.44	6.46	0.27	0	
			48	7.11	4.65	7.08	0.51	0.42	
			54	8.49	1.05	8.41	0.41	0.95	
			10	1.96	0.04	1.96	0.11	0	
			20	3.08	16.16	3.08	0.13	0	
				30	5.38	0.05	5.38	0.19	0

both uniformly distributed and clustered sets of demand requirements. These instances were solved using a maximum of 6 paths for both the metaheuristic procedure and the MIP formulation. The instance consisting of 17 nodes, 26 links, and 79 demands corresponds to a real instance. Table 4 displays the computational results obtained for a network with 50 nodes and 63 links for which four sets of demands have also been created. The instance consisting of 112 demands corresponds to a real instance.

In tables 3 and 4, columns 5 and 6 summarize costs and running times for the metaheuristic. Under the heading Cplex we have reported the total cost obtained by solving our MIP formulation and either the total solution time or the optimality gap, if Cplex cannot find an optimal solution after two hours of execution. Last column in both tables shows the deviation between Cplex solution and the metaheuristic solution. In the worst case, the deviation is no greater than 0.57% in table 3 and 2.73% in table 4 when Cplex is able to find and confirm the optimal solution. When Cplex fails to complete the branch and bound optimization, the upper bound solutions found are in all cases inferior to the solutions found with the metaheuristic procedure.

Specifically, in table 3, Cplex finds the optimal solution of the heuristic MIP model in five instances. For the other four instances, the execution of Cplex was stopped after two hours, obtaining inferior solutions than those obtained with the metaheuristic in a shorter time period. A similar pattern is observed in table 4, where Cplex fails twice to find the optimal solution within the 2-hour limit.

## 6 Conclusions

This paper presents a heuristic optimization model for the capacity expansion problem in WDM networks. We have carried out a comparative analysis be-

**Table 2.** Comparative results for networks with 12 nodes

Set Name	$ N $	$ E $	$ D $	<i>Metaheur</i> <i>Cost</i>	<i>Metaheur</i> <i>Time</i>	<i>Cplex</i> <i>Cost</i>	<i>Cplex</i> <i>Time</i>	$\frac{C_M - C_c}{C_c}$ (%)	
Extant0D	12	17	19	6.26	0.36	6.26	0.71	0	
			66	12.14	26.51	11.83	7.38	2.62	
			15	3.69	0.14	3.69	0.15	0	
			21	6.21	0.39	6.21	0.63	0	
			44	14.36	7.32	14.36	10.70	0	
			33	66	11.83	109.82	11.83	225.32	0
			15	3.69	0.43	3.69	1.41	0	
			21	7.32	3.45	7.32	10.94	0	
			44	14.23	38.17	13.94	81.16	2.08	
			46	66	11.77	152.12	11.77	224.90	0
			15	3.69	0.22	3.69	0.83	0	
			21	7.33	5.79	7.33	25.80	0	
			44	13.95	58.16	13.95	467.72	0	

**Table 3.** Comparative results for networks with 17 nodes

Set Name	$ N $	$ E $	$ D $	<i>Metaheur</i> <i>Cost</i>	<i>Metaheur</i> <i>Time</i>	<i>Cplex</i> <i>Cost</i>	<i>Cplex</i> <i>Time</i>	<i>Cplex</i> <i>gap</i> (%)	$\frac{C_M - C_c}{C_c}$ (%)	
Ex2D	17	26	79	176.96	185.10	180.08	-	3.14	-	
			135	172.94	65.18	173.04	-	1.97	-	
			28	81.84	0.95	81.84	13.42	-	0	
			27	23.22	2.14	23.22	5.99	-	0	
			81	97.37	97.63	96.81	296.16	-	0.57	
			68	68	66.81	42.12	69.35	-	1.41	-
			36	68.10	11.69	67.79	292.93	-	0.45	
			27	24.43	13.47	24.43	12.26	-	0	
			81	82.65	233.56	82.74	-	1.27	-	

tween the results obtained using a metaheuristic procedure for the problem on hand and the results obtained solving a mathematical model with Cplex. The mathematical model is solved as a relaxation of the original problem because we don't consider all possible segments or paths between each pair of demand requirements. Our experiments corroborate the effectiveness of the metaheuristic developed in [8] as the size of the problems increases. For relatively small problems (i.e., with number of nodes equal to 12 or less), solving the MIP formulation seems to be a better alternative than running the metaheuristic procedure.

An important consideration for additional research in this area deals with tackling uncertainty. Clearly, the demands cannot be considered known in an environment such as the telecommunications industry. The availability of a MIP formulation that can be used to find near-optimal solutions to the capacity

**Table 4.** Comparative results for networks with 50 nodes

Set Name	$ N $	$ E $	$ D $	<i>Metaheur</i> <i>Cost</i>	<i>Metaheur</i> <i>Time</i>	<i>Cplex</i> <i>Cost</i>	<i>Cplex</i> <i>Time</i>	<i>Cplex</i> <i>gap(%)</i>	$\frac{C_M - C_e}{C_e}$ (%)
National	50	63	112	42.88	230.70	42.88	262.41	-	0
			268	110.34	558.17	111.36	-	4.94	-
			91	59.13	138.08	59.43	-	1.65	-
			46	37.36	50.80	36.85	257.54	-	1.38
			66	51.09	112.76	49.73	1467.1	-	2.73

expansion problem represents a stepping-stone toward the solution of a stochastic version of the problem that treats the demands as uncertain.

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