

**MINI-COURSE**  
***Classical mechanics in spaces of constant curvature***  
**Mariano Santander**  
**Department of Theoretical Physics, Atomic Physics**  
**and Optics, University of Valladolid**

\* \*

7 and 9 February 2006  
Dep. de Matemàtica Aplicada IV, Campus Nord UPC, edifici C3, 204a  
Biblioteca de Matemàtica  
Tuesday 15:30-17:30, Thursday 11-13

**\*Abstract\***

In this course we present a framework that allows to work in the explicit description of both geometrical and mechanical characteristics of the classical motion in spaces of constant curvature with arbitrary signature. In the euclidian space, Kepler's potential and the harmonic oscillator turn out to be salient systems for their mathematical properties (related to the superintegrability) as well as for their physical relevance. In both cases the orbits are conics and the motion problem admits a complete analytical solution. Such remarkable feature remains unaltered when one replaces Euclidian space by any space of constant curvature. Both "curved" Kepler's potential and "curved" harmonic oscillator are superintegrable systems whose orbits are "conics" (in the sphere, the hyperbolic plane, ...).

A complete description of the conics in these spaces of constant curvature will be given, emphasising their visual representations. This information will be applied to describe completely the kind of orbits for both systems in spaces with curvature.

Finally we will comment on the well-known relationship between the euclidian Kepler motion and the geodesic flow in (velocity) spaces of constant curvature, and we will show that the same relationship applies, with minimal changes, when the motion occurs in a space of constant curvature.